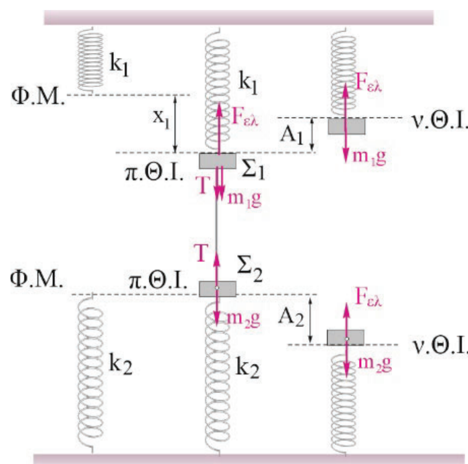
# Λύσεις κριτηρίου 12

**ΘΕΜΑ Α**

**Α**1. (α) **Α**2. (γ) **Α**3. (δ) **Α**4. (α) **Α5**. α. Λ β. Σ γ. Σ δ. Λ ε. Σ

**ΘΕΜΑ Β**

**Β1.** (**i**)




$$T_1 = T_2 \Rightarrow 2\pi\sqrt{\frac{m_1}{k_1}} = 2\pi\sqrt{\frac{m_2}{k_2}} \Rightarrow \frac{m_1}{k_1} = \frac{m_2}{k_2} \quad (1)$$

$$\text{Π.θ.ι.}: \Sigma F_2 = 0 \Rightarrow T = m_2 g \quad (2)$$

$$\text{Π.θ.ι.}: \Sigma F_1 = 0 \Rightarrow T + m_1 g = F_{\varepsilon\lambda} = k_1 x_1 \quad (3)$$

Από (2) και (3) $(m_1 + m_2)g = k_1 x_1 \quad (4)$

$$\text{Ν.θ.ι.}: \Sigma F_2 = 0 \Rightarrow m_2 g = F_{\varepsilon\lambda} = k_2 x_2$$

$$\Rightarrow x_2 = A_2 = \frac{m_2 g}{k_2}$$

$$\text{Ν.θ.ι.}: \Sigma F_1 = 0 \Rightarrow$$

$$m_1 g = F_{\varepsilon\lambda} = k_1(x_1 - A_1) = k_1 x_1 - k_1 A_1 \Rightarrow$$

$$m_1 g = (m_1 + m_2)g - k_1 A_1 \Rightarrow A_1 = \frac{m_2 g}{k_1}$$

$$A_1 = A_2 \Rightarrow \frac{m_2 g}{k_1} = \frac{m_2 g}{k_2} \Rightarrow k_1 = k_2 \xrightarrow{(1)} m_1 = m_2$$

**Β2.** (**i**)

$$W_{F_{\varepsilon\pi}} = -\Delta U = U_B - U_\Gamma = \frac{1}{2}D{x_B}^2 - \frac{1}{2}D{x_\Gamma}^2 \quad (1)$$

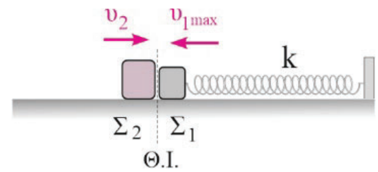
$$\text{ΑΔΕΤ}: E = K_B + U_B \Rightarrow \frac{1}{2}DA^2 = K_B + \frac{1}{2}D{x_B}^2 \Rightarrow K_B = \frac{1}{2}D\left(A^2 - {x_B}^2\right)$$

Ομοίως $K_\Gamma = \frac{1}{2}D\left(A^2 - {x_\Gamma}^2\right)$

$$\frac{K_B}{K_\Gamma} = \frac{\frac{1}{2}D\left(A^2 - {x_B}^2\right)}{\frac{1}{2}D\left(A^2 - {x_\Gamma}^2\right)} \Rightarrow \frac{A^2 - 3{x_\Gamma}^2}{A^2 - {x_\Gamma}^2} = \frac{1}{3} \Rightarrow x_\Gamma = \pm\frac{A}{2} \quad \text{και} \quad x_B = \pm\frac{A\sqrt{3}}{2}$$

Από (1) $W_{F_{\varepsilon\pi}} = \frac{1}{2}D\frac{3A^2}{4} - \frac{1}{2}D\frac{A^2}{4} \Rightarrow W_{F_{\varepsilon\pi}} = \frac{E}{2}$

**Β3.** **(iii)**




$$\alpha_{1,max} = \alpha_{2,max} \Rightarrow \omega_1^2 A_1 = \omega_2^2 A_2 \Rightarrow$$

$$\frac{k}{m_1} A_1 = \frac{k}{m_1 + m_2} A_2 \Rightarrow A_2 = 4A_1$$

$$\text{ΑΔΟ}: P_{\alpha\rho\chi} = P_{\tau\varepsilon\lambda} \Rightarrow m_2\upsilon_2 - m_1\upsilon_{1,max} = (m_1 + m_2)V_{max} \Rightarrow$$

$$3m\upsilon_2 - m\omega_1 A_1 = 4m\omega_2 A_2 \Rightarrow 3m\upsilon_2 - m\sqrt{\frac{k}{m_1}}A_1 = 4m\sqrt{\frac{k}{m_1 + m_2}}A_2 \Rightarrow \upsilon_2 = 3A_1\sqrt{\frac{k}{m_1}}$$

**ΘΕΜΑ Γ**

**Γ1.**

$$\text{ΑΔΕΤ}: E = K + U \Rightarrow K = E - \frac{1}{2}Dx^2 = 1 - 25x^2 \Rightarrow D = 50N/m \ \ \text{και} \ \ E = 1J$$

$$T = 2\pi\sqrt{\frac{m}{D}} \Rightarrow T = \frac{\pi}{5}s$$

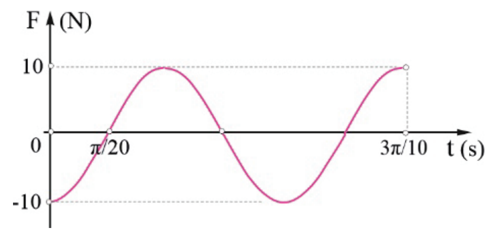
**Γ2.** $E = \frac{1}{2}DA^2 \Rightarrow A = 0,2m$ , $\omega = \frac{2\pi}{T} = 10rad/s$

Για t=0 είναι χ=Α,

$$x = A\eta\mu(\omega t + \varphi_0) \Rightarrow A = A\eta\mu\varphi_0 \Rightarrow \varphi_0 = \frac{\pi}{2}rad$$

$$F = -Dx = -DA\eta\mu\left(\omega t + \frac{\pi}{2}\right) = -10\eta\mu\left(10t + \frac{\pi}{2}\right) \quad \text{(S.I.)},$$

$$\Delta t = \frac{3\pi}{10}s = 1,5T$$




**Γ3.** $\upsilon_1 = \omega A\sigma\upsilon\nu\left(\omega t_1 + \frac{\pi}{2}\right) = 2\sigma\upsilon\nu\left(10t_1 + \frac{\pi}{2}\right) = -1m/s$

$$\upsilon_2 = \omega A\sigma\upsilon\nu\left(\omega t_2 + \frac{\pi}{2}\right) = 2\sigma\upsilon\nu\sigma\upsilon\nu\left(10t_2 + \frac{\pi}{2}\right) = 2m/s$$

$$\Delta p = p_2 - p_1 = m\upsilon_2 - m\upsilon_1 \Rightarrow \Delta p = 1,5kgm/s$$

**Γ4.** $x_1 = A = 0,2m$ , $x_2 = A\eta\mu\left(\omega t_2 + \frac{\pi}{2}\right) = 0,2\eta\mu\left(10t_1 + \frac{\pi}{2}\right) = 0$

$$t_2 = \frac{7\pi}{20}s = \frac{7}{4}T, \quad d = 7A = 1,4m$$

**Γ5.** $x_1 = A\eta\mu\left(\omega t_1 + \frac{\pi}{2}\right) = 0,2\eta\mu\left(10t_1 + \frac{\pi}{2}\right) = -0,1\sqrt{3}m$

$$\frac{dK}{dt} = \frac{W_{F_{\varepsilon\pi}}}{\Delta t} = F_{\varepsilon\pi}\upsilon_1 = -Dx_1\upsilon_1 \Rightarrow \frac{dK}{dt} = -5\sqrt{3}J/s$$

**ΘΕΜΑ Δ**

**Δ1.** $\Pi.\theta.\iota.: \Sigma F = 0 \Rightarrow N + mg = F_{\varepsilon\lambda} = 10N$

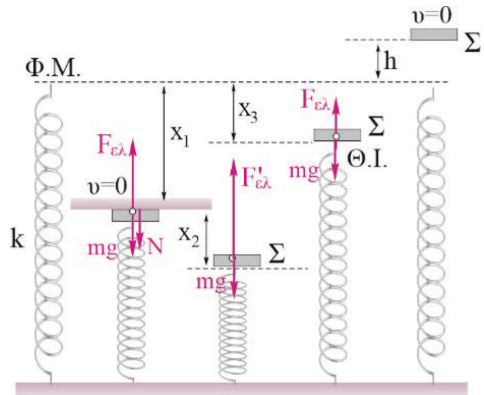
$F_{\varepsilon\lambda} = kx_1 \Rightarrow x_1 = 0{,}1m$ ,

$F_{\varepsilon\lambda}' = 1{,}5F_{\varepsilon\lambda} = 15N = k(x_1 + x_2) \Rightarrow x_2 = 0{,}05m$

$N.\theta.\iota.: \Sigma F = 0 \Rightarrow mg = F_{\varepsilon\lambda} = kx_3 \Rightarrow$

$x_3 = \frac{mg}{k} = 0{,}05m$

$A = x_1 + x_2 - x_3 \Rightarrow A = 0{,}1m$




**Δ2.** $\omega = \sqrt{\frac{k}{m}} \Rightarrow \omega = 10\sqrt{2} rad/s$

Για t=0 είναι x=A, άρα

$$x = A\eta\mu(\omega t + \varphi_0) \Rightarrow A = A\eta\mu\varphi_0 \Rightarrow \varphi_0 = \frac{\pi}{2} rad$$

$$x = A\eta\mu\left(\omega t + \frac{\pi}{2}\right) = 0{,}1\eta\mu\left(10\sqrt{2}t + \frac{\pi}{2}\right) \Rightarrow x_1 - x_3 = 0{,}05 = 0{,}1\eta\mu\left(10\sqrt{2}t + \frac{\pi}{2}\right) \Rightarrow$$

$$\eta\mu\left(10\sqrt{2}t + \frac{\pi}{2}\right) = 0{,}5 = \eta\mu\frac{\pi}{6} \xrightarrow{\upsilon\langle 0} 10\sqrt{2}t + \frac{\pi}{2} = \frac{5\pi}{6} \Rightarrow t = \frac{\pi\sqrt{2}}{60} s$$

**Δ3.**

$$A\Delta E T: \frac{1}{2}kA^2 = \frac{1}{2}m\upsilon^2 + \frac{1}{2}k(x_1 - x_3)^2 \Rightarrow \upsilon = \pm\sqrt{\frac{3}{2}} m/s \Rightarrow \upsilon = -\sqrt{\frac{3}{2}} m/s$$

$$\frac{dU_{\varepsilon\lambda}}{dt} = -\frac{W_{F_{\varepsilon\lambda}}}{\Delta t} = -F_{\varepsilon\lambda}\upsilon = -(-kx_1)\upsilon \Rightarrow \frac{dU_{\varepsilon\lambda}}{dt} = -10\sqrt{\frac{3}{2}} J/s$$

**Δ4**. Η επαφή του σώματος από το ελατήριο θα χαθεί όταν αυτό φτάσει στο φυσικό μήκος του.

$$A\Delta E T: \frac{1}{2}kA^2 = \frac{1}{2}m\upsilon_2^2 + \frac{1}{2}kx_3^2 \Rightarrow \upsilon_2 = \pm\sqrt{\frac{3}{2}} m/s$$

$$A\Delta M E: K_{\alpha\rho\chi} = U_{\tau\varepsilon\lambda} \Rightarrow \frac{1}{2}m\upsilon_2^2 = mgh \Rightarrow h = 0{,}075m$$ ,

$$H = x_1 + h = 0{,}175m$$